

Class 8 - Chapter 1: Rational Numbers

Quick Revision Notes for Exam

1. Introduction

Rational numbers are needed to solve equations that require solutions beyond whole numbers and integers.

- Rational number = $\frac{p}{q}$ where p and q are integers and $q \neq 0$
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2. Closure Property

Definition

A set is **closed** under an operation if performing that operation on elements of the set always gives a result in the same set.

Closure Table

| Numbers | Addition | Subtraction | Multiplication | Division |
|------------------|----------|-------------|----------------|----------|
| Whole Numbers | Yes | No | Yes | No |
| Integers | Yes | Yes | Yes | No |
| Rational Numbers | Yes | Yes | Yes | No* |

Table 1: Closure Property

Note: Rational numbers are closed under division except when dividing by zero.

3. Commutativity Property

Definition

An operation is **commutative** if changing the order of operands doesn't change the result.

- Formula: $a \circ b = b \circ a$

Commutativity Table

| Numbers | Addition | Subtraction | Multiplication | Division |
|------------------|----------|-------------|----------------|----------|
| Whole Numbers | Yes | No | Yes | No |
| Integers | Yes | No | Yes | No |
| Rational Numbers | Yes | No | Yes | No |

Table 2: Commutativity Property

Examples for Rational Numbers:

- Addition: $\frac{-2}{3} + \frac{1}{7} = \frac{1}{7} + \frac{-2}{3}$ ✓
- Subtraction: $\frac{2}{5} - \frac{3}{4} \neq \frac{3}{4} - \frac{2}{5}$ ✗
- Multiplication: $\frac{-7}{5} \times \frac{6}{3} = \frac{6}{3} \times \frac{-7}{5}$ ✓

4. Associativity Property

Definition

An operation is **associative** if grouping of operands doesn't change the result.

- Formula: $a \circ (b \circ c) = (a \circ b) \circ c$

Associativity Table

| Numbers | Addition | Subtraction | Multiplication | Division |
|------------------|----------|-------------|----------------|----------|
| Whole Numbers | Yes | No | Yes | No |
| Integers | Yes | No | Yes | No |
| Rational Numbers | Yes | No | Yes | No |

Table 3: Associativity Property

Example for Addition:

$$\left(-\frac{2}{3} + \frac{3}{5}\right) + \left(-\frac{7}{6}\right) = -\frac{2}{3} + \left(\frac{3}{5} + \left(-\frac{7}{6}\right)\right)$$

5. Identity Elements

Additive Identity

- 0 is the additive identity for rational numbers
- $a + 0 = 0 + a = a$ for any rational number a

Multiplicative Identity

- 1 is the multiplicative identity for rational numbers
- $a \times 1 = 1 \times a = a$ for any rational number a

6. Distributivity Property

Multiplication is **distributive** over addition and subtraction.

Formula

For all rational numbers a , b , and c :

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

Example

$$-\frac{3}{4} \times \left(\frac{2}{3} + \frac{5}{6} \right) = \left(-\frac{3}{4} \times \frac{2}{3} \right) + \left(-\frac{3}{4} \times \frac{5}{6} \right)$$

7. Key Formulas Summary

| Property | Formula |
|--------------------------------|---|
| Closure | $a + b, a - b, a \times b$ all rational |
| Commutativity (Addition) | $a + b = b + a$ |
| Commutativity (Multiplication) | $a \times b = b \times a$ |
| Associativity (Addition) | $a + (b + c) = (a + b) + c$ |
| Associativity (Multiplication) | $a \times (b \times c) = (a \times b) \times c$ |
| Additive Identity | $a + 0 = a$ |
| Multiplicative Identity | $a \times 1 = a$ |
| Distributivity | $a(b + c) = ab + ac$ |

Table 4: Properties of Rational Numbers

8. Quick Checklist for Exam

- ✓ Rational numbers are closed under addition, subtraction, and multiplication
 - ✓ Addition and multiplication are both commutative for rationals
 - ✓ Subtraction and division are NOT commutative
 - ✓ Addition and multiplication are both associative for rationals
 - ✓ Subtraction and division are NOT associative
 - ✓ Zero is additive identity; One is multiplicative identity
 - ✓ Distributivity applies to multiplication over addition/subtraction
 - ✓ Division by zero is not allowed
 - ✓ Every property table must be memorized with Yes/No answers
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9. Important Definitions

Rational Number: A number of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$

Closure: A set is closed under an operation if the result of the operation is always in the set

Commutative: Order doesn't matter: $a \circ b = b \circ a$

Associative: Grouping doesn't matter: $a \circ (b \circ c) = (a \circ b) \circ c$

Identity Element: A special element that doesn't change other elements (0 for addition, 1 for multiplication)

Distributive: Multiplication distributes over addition: $a(b + c) = ab + ac$

References

[1] NCERT. (2024-25). Mathematics Textbook for Class 8, Chapter 1: Rational Numbers.